Design Optimization of Linear Synchronous Motors for Overall Improvement of Thrust, Efficiency, Power Factor and Material Consumption

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Abstract

By having accurate knowledge of the magnetic field distribution and the thrust calculation in linear synchronous motors, assessing the performance and optimization of the motor design are possible. In this paper, after carrying out a performance analysis of a single-sided wound secondary linear synchronous motor by varying the motor design parameters in a layer model and a d-q model, machine single- and multi-objective design optimizations are carried out to improve the thrust density of the motor based on the motor weight and the motor efficiency multiplied by its power factor by defining various objective functions including a flexible objective function. A genetic algorithm is employed to search for the optimal design. The results confirm that an overall improvement in the thrust mean, efficiency multiplied by the power factor, and thrust to the motor weight ratio are obtained. Several design conclusions are drawn from the motor analysis and the design optimization. Finally, a finite element analysis is employed to evaluate the effectiveness of the employed machine models and the proposed optimization method.

Key Words: Finite element analysis, Electromagnetic fields, Linear synchronous motors, Modelling, Optimization

I. INTRODUCTION

Linear synchronous motors (LSMs) enjoy high efficiency due to a lack of slip losses and a high magnetizing current. Also, the machine power factor can be controlled to higher values than a fixed power factor which is obtained with a comparable induction motor at the same output power and speed. A higher efficiency and power factor leads to a significant reduction in the inverter rating, resulting in a substantial cost saving. With an effective design optimization for LSMs, the benefits can be emphasized to varying degrees.

Many aspects of LSMs have been studied in the literature, including their modeling, analysis, design, control and optimization. Among these studies machine modeling plays a fundamental role since it is required for all other studies [1]-[4]. The design optimization of the motors is also important from a practical point of view since it reduces both the machine’s primary and operating costs. It is carried out based on machine models given accurate knowledge of magnetic field distribution, thrust, efficiency, power factor, etc. Among the limited work on the design optimization of wound secondary linear synchronous motors (WSLSMs), the optimization of the secondary electromagnet shape has been investigated by using a finite element method (FEM) based model of the motor [5].

However, a FEM based optimization is appropriate for the final stages of design optimization, is not time efficient for the iterative procedures in the pre-final stages. Alternatively, design optimizations based on analytical models are executed rapidly in comparison with FEM based optimizations. These optimization methods are necessary as the main tool to cope with the repeated runs of optimization procedures. However, analytical model based optimizations have focused heavily on permanent magnet linear synchronous motors (PMLSMs) rather than WSLSMs. Different objective functions and many design optimization variables are selected and studied for the optimization of such machines [6]–[11]. However, PMLSMs are mainly used in low power automation applications. They are still not considered for use in high power high speed applications like maglev.

In this paper a design optimization for WSLSMs based on an analytical model is presented. The thrust density of the motor based on its weight and the power factor multiplied by its efficiency are the two objective functions chosen for this research. These functions are chosen since \( \eta \cos \varphi \) is an important machine characteristic for the inverter rating in motor design, and the thrust density is an essential factor in evaluating the motor performance. In fact, the thrust mean, weight, efficiency and power factor of a WRLSM are improved based on a machine layer model and a d-q electrical model. The machine DC excitation pole dimensions, the pole pitch, the tooth width, the motor width, the primary current density,
the ratio of the $d$-axis to the $q$-axis component of the primary current and the yoke height are chosen as design variables. A flexible objective function is defined including the thrust to weight ratio, the efficiency and the power factor. A genetic algorithm (GA) optimization is then carried out to find out the best set of design variables. Finally, a finite element analysis is employed to verify the optimization results.

II. MACHINE MODEL

A. Motor Topology

Fig. 1 shows an upside down schematic view of a single-sided WSLSM with a long primary. The primary includes a three-phase iron-core winding which is extended along the motion path. The slot-openings of the windings total three slots. Three phases are supplied from several inverter stations which are placed along the path.

The moving part of the motor is a short secondary which consists of a back iron and DC excitation electromagnetic poles. The secondary is located above the primary with an air gap, $g$, and should experience a contactless motion along the path. The electromagnetic poles of the secondary consist of DC excited copper windings and iron cores which are connected to the back iron.

The parameters and dimensions of the motor are as follow: $\tau_s$ is the slot pitch, $h_s$ is the slot width, $h_m$ is the electromagnetic pole height, $\tau_p$ is the pole pitch, and $\tau_m$ is the electromagnet width which is the width of the iron core of a pole plus almost half of the DC coil width, $h_l$ is the slot height, and $L$ is the motor width along the $z$ direction.

B. Analytical Model

In this subsection a method for developing a layer model for the motor presented in the previous subsection is briefly recalled [12]. All of the harmonics of the magnetic field produced by the DC-excited electromagnets are considered. The method is based on solving a Maxwell equations in several layers of the motor. The layer representing the DC excitation is modeled by an anisotropic layer with different permeability values along the $x$ and $y$ directions as follows, where the $x$-axis is along the motor length and the $y$-axis is along the air gap [12]:

$$\mu_{xm} = \mu_0 \left[1 + \frac{\mu_r}{\tau_p - \tau_m + \mu_r (1 - (\tau_p - \tau_m)/\tau_p)}\right]$$

$$\mu_{ym} = \mu_0 \left[1 + \frac{\mu_r}{\tau_p - \tau_m + \mu_r (1 - (\tau_p - \tau_m)/\tau_p)}\right]$$

and $\mu_r$ is the relative permeability of iron.

The Maxwell equations lead to Laplace and Poisson equations. It is known that the flux density in each layer can be achieved by the curl of the magnetic vector potential of the layer. As a result, the normal component of the flux density at $y = y_0$ in the air gap is given by [12]:

$$B_y(x) = \frac{\partial A}{\partial x} = -\sum_{n=1,3,...}^{\infty} \frac{n\pi}{\tau_p} \left(C_1 e^{\frac{2\pi n y}{\tau_p}} + C_2 e^{-\frac{2\pi n y}{\tau_p}}\right) \cos \left(\frac{n\pi x}{\tau_p}\right)$$

where $C_1 - C_2$ can be found as:

$$C_1 = \frac{k(n)}{\left(1 + e\left(\sum_{n=1}^{\infty} \frac{\mu_n J(n)}{n^2 \pi^2} \frac{\tau_p^2}{\tau_m}\right)\right)}$$

$$C_2 = C_1 e^{\frac{2\pi n y}{\tau_p}} g e$$

and $J(n)$ is the maximum of the equivalent current density of the electromagnetic pole given by:

$$J(n) = \frac{4}{\tau_p} \frac{N_{dc} I_{dc}}{h_m} \sin \left(\frac{nn\pi}{2}\right)$$

where $\eta$ stands for the electromagnet width to pole pitch ratio.

C. Machine Characteristics

The flux density obtained in the previous subsection along with an electrical model of the machine is employed to calculate the motor characteristics including the developed thrust, the efficiency, the power factor, etc. A conventional $d$-$q$ electrical model of the machine in a synchronously rotating reference frame is used here. In this model, the flux distribution in the air gap — which is only produced by the electromagnetic poles — is assumed to be sinusoidal and the magnetic saturation is not considered. The motor thrust is then obtained as [7]:

$$F_{x(av)} = \frac{3}{2} \pi \frac{\lambda_{DC\text{-excitation}}}{\tau_p} \left(L_d - L_q\right) \times i_d \times i_q$$

In the motor considered in this paper $L_d$ is not equal to $L_q$, but for $i_d = 0$ the second term of the thrust vanishes, so (6) is simplified to:

$$F_{x(av)} = \frac{3}{2} \pi \frac{\lambda_{DC\text{-excitation}}}{\tau_p} i_q = 3I_{ph} \times (B_{1g} L N_{ph} k_{w1})$$

where $B_{1g}$ is the maximum of the fundamental harmonic of the field in the air gap which is obtained from (2), $I_{ph}$ is the maximum of the primary current, and $S_1$ is the maximum of the equivalent line current density of the stator. The direct axis current, $i_d$, is considered as a variable parameter.

In this kind of motor, the iron loss of the secondary is negligible. Therefore, the iron loss is due to the primary ($P_{iron}$)
which includes the eddy current, the hysteresis losses of the teeth and the stator yoke which are obtained as follows [13]:

\[
\begin{align*}
P_{\text{eddy(teeth)}} &= \frac{16k_{\text{eddy}}\gamma f^2 B_{\text{y}}^2}{w_t} V_{\text{teeth}} \\
P_{\text{eddy(yoke)}} &= \frac{32k_{\text{eddy}}\gamma f^2 B_{\text{y}}^2}{w_y} V_{\text{yoke}} \\
P_{\text{hyst(teeth)}} &= 2\pi f k_{\text{hyst}} B_{\text{y}}^2 V_{\text{teeth}} \\
P_{\text{hyst(yoke)}} &= 2\pi f k_{\text{hyst}} B_{\text{y}}^2 V_{\text{yoke}}
\end{align*}
\]  

(8)

where \(k_{\text{eddy}}\) is the coefficient of the eddy current losses which is a value between 0.04-0.07, according to the material characteristics of iron and its lamination, \(k_{\text{hyst}}\) is the coefficient of the hysteresis losses which is a value between 1.8-2, \(V_{\text{teeth}}\) is the total volume of the stator teeth and \(V_{\text{yoke}}\) is the total volume of the stator yoke.

The other parts of the electrical loss are the copper losses of the primary and the secondary which are expressed as:

\[
P_{\text{cu}(p)} = 3R_p I_p^2 \\
P_{\text{cu}(s)} = 2PR_s I_s^2
\]

(9, 10)

where \(R_p\) and \(R_s\) are the resistances of the primary and the secondary windings respectively and \(I_f\) is the DC current of the secondary. Also, there are additional losses \(P_{\text{add}}\), including the mechanical loss and the stray loss. Therefore, the motor efficiency is given by:

\[
\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{iron}} + P_{\text{cu}} + P_{\text{add}}}
\]

(11)

where \(P_{\text{out}} = F_{\text{out}}\), \(V_s\) and \(V_t\) are the synchronous speed. Considering the \(d\)-axis current of the stator to be equal to zero, the power factor of the motor is obtained as follows:

\[
\cos \varphi = \frac{E_f + R_p I_q}{V_t}
\]

(12)

where \(I_q\) is the stator current along the \(q\)-axis and \(E_f\) and \(V_t\) are the back EMF and the terminal voltage of the stator, respectively which are obtained as [14]:

\[
E_f = 4.44f k_{\text{w1}} \Phi_s N_{ph}
\]

(13)

\[
V_t = \sqrt{(E_f - I_q X_{id})^2 + (I_q X_{iq})^2}
\]

(14)

where \(f\) is the frequency, \(k_{\text{w1}}\) is the armature winding coefficient \(\Phi_s\) is the airgap magnetic flux, \(N_{ph}\) is the number of the armature turns per phase and \(X_{id}\) and \(X_{iq}\) are the \(d\)-axis and \(q\)-axis armature reaction reactances respectively.

III. Optimization Problem

An optimization problem with \(p\) objectives, \(n\) variables, and \(m\) constraints is formulated as

\[
\text{Maximize } f_1(x), f_2(x), \ldots, f_p(x) \quad x \in K
\]

(15)

where \(x \in \mathbb{R}^n\) and \(f : \mathbb{R}^n \to \mathbb{R}\). Also, \(K\) is a feasible set of solutions (15) which are described by

\[
K = \{x \in \mathbb{R}^n : g_i(x) \leq 0, i = 1, 2, \ldots, p\}
\]

(16)

The constraints \(g_i(x)\) limit the design variables. The design variables are chosen as the motor width, the electromagnet height, the electromagnet width to pole pitch ratio, the pole pitch, the yoke height, the primary current density \((I_p)\), the ratio of the \(d\)-axis to \(q\)-axis component of the primary current and the tooth width, based on their importance as influences on the optimization objectives as considered later in this section. The fixed variables are the slot pitch, the air gap, and the secondary windings current density according to their rather less important influences on the objectives.

The design objectives in this paper are to increase the motor developed thrust, the efficiency (\(\eta\)) and the power factor (\(\cos \varphi\)), and to reduce the motor weight. However, while copper is more expensive than iron, the iron lamination is also expensive. So to have a cost effective optimization the both copper weight and the iron weight are considered. The objectives improve the most important aspects of the motor performance and cost. For improving the efficiency and power factor we consider the objective function as \(\eta \cos \varphi\), and for improving the motor thrust and reducing its weight the objective function of the thrust to weight ratio is selected.

Since the length of the secondary is limited and the stator of the motor is as long as the motion path, the weight of the motor per meter \((w)\) instead of the total weight of the motor is considered as a design variable. The weight of the motor per unit of length is calculated as:

\[
w = (V_{cu-p} \times \rho_{cu} + V_{i-p} \times \rho_i + V_{cu-s} \times \rho_{cu} + V_{i-s} \times \rho_i)/(p \tau_p)
\]

(17)

where \(V_{cu-p}\), \(V_{cu-s}\), \(V_{i-p}\) and \(V_{i-s}\) are the primary and secondary copper volumes and the primary and secondary iron
Fig. 4. Variations of thrust to weight ratio and $\eta \cos \varphi$ with the tooth width.

Fig. 5. Variations of thrust to weight ratio and $\eta \cos \varphi$ with the yoke height.

Fig. 6. Variations of thrust to weight ratio and $\eta \cos \varphi$ with the pole pitch.

Fig. 7. Variations of thrust to weight ratio and $\eta \cos \varphi$ with primary current density.

Fig. 8. Variations of thrust to weight ratio and $\eta \cos \varphi$ with the ratio of the $d$-axis to $q$-axis component of the primary current.
Design Optimization of Linear Synchronous Motors . . .

Fig. 8. Variations of thrust to weight ratio and $\eta \cos \phi$ with the motor width.

Fig. 9. Variations of thrust to weight ratio and $\eta \cos \phi$ with the ratio of the $d$-axis to $q$-axis components of primary current.

an objective function provides a higher degree of freedom in selecting the appropriate motor parameters.

A number of constraints can also be taken into account during the optimization to prevent the possibility of reaching unrealistic optimization results. The motor width is limited by an upper bound to prevent low efficiency and by a lower bound to reduce leakage flux effect. The lower bound of the electromagnet height is dependent on the placing of DC excitation windings around it. The tooth width and the yoke height are limited by the lower bounds to prevent saturation. The primary current density bounds depend on the thermal constraints. Since the motor efficiency, the thrust and the power factor increase with an increase in motor size, limiting the upper bound of the motor weight is beneficial. Finally, the lower bound of the thrust is limited to the minimum required thrust. The limiting values of the design variables are listed in Table I.

![Table I](image)

**Table I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole Height</td>
<td>m</td>
<td>$h_{em}$</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Electromagnet Width to Pole Pitch Ratio</td>
<td>-</td>
<td>$r_m/\tau_p$</td>
<td>0.5</td>
<td>0.9</td>
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<tr>
<td>Tooth Width</td>
<td>m</td>
<td>$w_s$</td>
<td>0.055</td>
<td>0.076</td>
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<tr>
<td>Primary Current Density</td>
<td>A/mm²</td>
<td>$J_p$</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Yoke Height</td>
<td>m</td>
<td>$h_y$</td>
<td>0.135</td>
<td>0.235</td>
</tr>
<tr>
<td>Weight</td>
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<td>$w$</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>Pole Pitch</td>
<td>m</td>
<td>$\tau_p$</td>
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<td>0.3</td>
</tr>
<tr>
<td>The Ratio of $d$-axis to $q$-axis Currents</td>
<td>-</td>
<td>$x$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Thrust</td>
<td>kN</td>
<td>$F_t$</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>Motor Width</td>
<td>m</td>
<td>$L$</td>
<td>0.5</td>
<td>13.5</td>
</tr>
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</table>

IV. DESIGN OPTIMIZATION

Different design optimizations are carried out in this section depending on the selected objectives. The first optimization is aimed toward a maximization of the thrust to weight ratio. The next optimization is concerned with the maximization of the efficiency multiplied by the power factor. Finally, all the objectives are integrated into a single objective function. The general form of the objective function defined in (20) provides an opportunity to perform all of these optimizations according to the same procedure by choosing appropriate values for $a$ and $b$. A genetic algorithm (GA) is employed to find the optimal design in each optimization. However, it will be shown that for finding the optimal design in the thrust to weight ratio optimization a GA is not required.

A. Genetic Algorithm

A GA provides a random search technique to find the global optimal solution in a complex multidimensional search space. The algorithm consists of three basic operators, i.e. the selection, the crossover, and the mutation. First, an initial population is produced randomly. Then, the genetic operators are applied to the population to gradually improve their fitness. This procedure yields a new population in each iteration. In this paper, the Roulette wheel method and the Tournament method with equal probabilities are used for the selection of the parents’ chromosomes [17]. The GA parameters used in this paper are shown in Table II.

![Table II](image)

**Table II**

<table>
<thead>
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<th>Parameter</th>
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<tbody>
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<td>Probability of Tournament</td>
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<tr>
<td>Probability of Roulette wheel</td>
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<tr>
<td>Probability of crossover</td>
<td>0.93</td>
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<tr>
<td>Probability of mutation</td>
<td>0.07</td>
</tr>
<tr>
<td>Population size</td>
<td>12</td>
</tr>
<tr>
<td>Number of Generations</td>
<td>40</td>
</tr>
</tbody>
</table>

B. Maximization of the Thrust to Weight Ratio

In this optimization, the objective function is defined as $F_t/w$. In this case the maximization of the thrust to weight ratio is aimed with an equal emphasis on thrust and weight. It can be concluded from Figs. 3-9 that for maximizing the thrust and minimizing the weight, most of the optimization problem take their boundary values. For example, the yoke,
the electromagnet heights and the pole pitch should be set to their minimum and the primary current density should be set to its maximum. Only the parameters of the tooth width and the electromagnet width to pole pitch ratio take values between their boundaries for optimization purposes. In this case, it is not necessary to use a GA for finding the optimal design. Since the number of variables is reduced to two variables, a 3-D figure is useful to give the optimal design. Fig. 10 shows the variations in the objective function of the thrust to weight ratio with the tooth width and the electromagnet width. The dimensions and characteristics of the optimized motor are shown in Table III as optimized motor1. In comparison with the original motor, in the optimized motor the ratio of thrust to weight increases from 1.276 to 1.427, i.e. a 12% improvement. The motor losses are also calculated for both the optimized and the original motor, in the optimized motor the ratio of thrust to weight increases from 1.276 to 1.427, i.e. a 12% improvement. As a result, an overall optimization is achieved with this optimization.

V. DESIGN EVALUATION

The design optimizations in this work were carried out based on the analytical machine model presented in Section II. Therefore, validity of the design optimizations depends greatly on the accuracy of this model. However, the model is obtained through some simplifications such as ignoring the saturation and considering an unlimited motor length. Thus, it is necessary to evaluate the extent of the model accuracy. In this section, a 2-D finite element analysis is employed to evaluate the model. It is supposed that the motor is controlled with a current-controlled inverter. The forces are then calculated using the local virtual work method. Numerical and graphical results are obtained. Fig. 11 shows a graphical representation of the flux lines in the WRLSM. The thrusts for different designs are obtained by the FEM and are compared with those obtained by the analytical model as in Table III. It is seen that the results of the analytical model are very close to the results of the FEM. The maximum error in the case of the thrust calculation is less than 4%, which is reasonable. This proves the validity of the proposed analytical model based design optimization.

VI. CONCLUSIONS

Different design optimizations are performed on a WRLSM to achieve a high developed thrust, a reduced motor weight,
and a high $\eta \cos \phi$. A layer model and a $d$-$q$ model of the machine are used in defining the optimization problem in each case. Low efficiency is avoided by limiting the motor width by an upper bound. The machine’s DC excitation pole dimensions, the pole pitch, the tooth width, the motor width, the primary current density, the ratio of the $d$-axis to $q$-axis component of the primary current and the yoke height are found to be appropriate design variables due to their rather important influence on the objective functions.

A genetic algorithm is employed to search for the optimal design variables. By considering a lower bound constraint for the motor developed thrust, the thrust to weight ratio and $\eta \cos \phi$ are optimized independently to 12% and 7.8% of those of the original motor, respectively. These two optimizations confirm the necessity of an overall design optimization taking into account both the thrust to weight ratio and $\eta \cos \phi$ simultaneously. The results of such an optimization show that it can increase both the thrust to weight ratio and $\eta \cos \phi$. Although, a slight increase in the motor losses can be observed, the results show an overall improvement in the objectives and they are clearly better than the results of the previous two optimizations.

Finally a finite element analysis confirms both the validity of the analytical model used and the performed design optimizations. However, taking into account the specific phenomena of the machine such as the end effects and the rather large air gap, an experimental evaluation of the machine would be useful for further confirmation of the optimization and this is recommended as a future step.

REFERENCES


